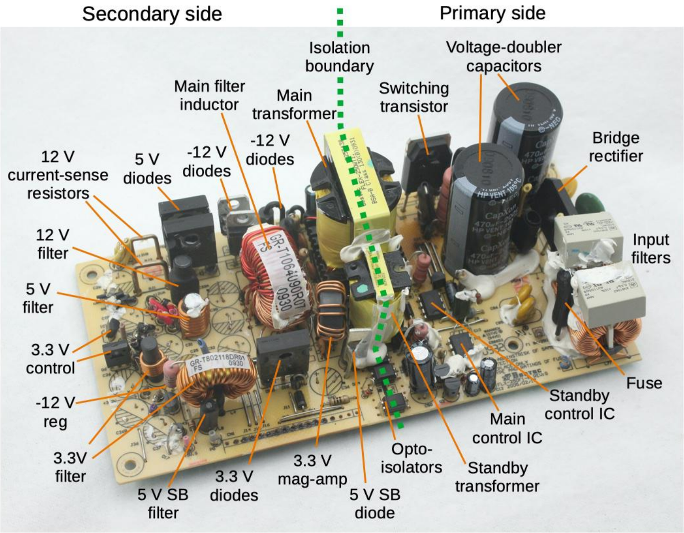

Finite output resistance in saturation

Quick review

The equation used to define iD depends on relationship between v_{DS} and v_{OV}:

- When $v_{DS} \ll v_{OV}$ (i.e., the small v_{DS} model)

$$i_D = \left[(\mu_n C_{ox} \left(\frac{W}{L} \right) v_{OV}) \right] v_{DS} \tag{1}$$

- When $v_{DS} < v_{OV}$ (i.e., the large v_{DS} model)

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Finite output resistance in saturation

- In previous lectures, we assume (in saturation) iD is independent of v_{DS}.
- Therefore, a change in v_{DS} has no effect on i_D.
    - This implies that the incremental resistance R_S is infinite
    - It is based on the idealization that, once the n-channel is pinched off, changes in v_{DS} will have no effect on i_D.
    - The problem is that, in practice, this is not completely true.
- In reality, the drift current increases, and i_D increases with increasing v_{DS}

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Finite Output Resistance in Saturation

What effect does increasing v_{DS} has on the n-channel once pinch-off has occurred?

- It will cause the pinch-off point to move slightly away from the drain and create new depletion region.
- Voltage across the (now shorter) channel will remain at v_{OV}.
- However, the additional voltage applied at v_{DS} will be seen across the "new" depletion region.

Finite Output Resistance in Saturation

What effect will increased v_{DS} has on n-channel once pinch-off has occurred?

- This voltage accelerates electrons as they reach the drain end, and sweep them across the "new" depletion region.
- However, at the same time, the length of the n-channel will decrease. This is known as channel length modulation.

Finite output resistance in saturation

- When $v_{DS} > V_{OV}^2$, the depletion region around the drain region grows in size.
- With depletion-layer widening, the channel length is in effect reduced, from L to $L - \Delta L$, a phenomenon known as channel-length modulation.

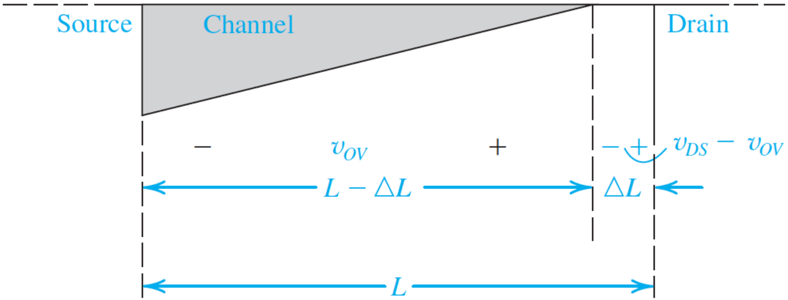

FIG 1. Early Effect—Finite Output Resistance
increasing v_{DS} beyond v_{DSsat} causes the channel pinch-off point to move slightly away from the drain; thus, reducing the effective channel lengthy by ΔL

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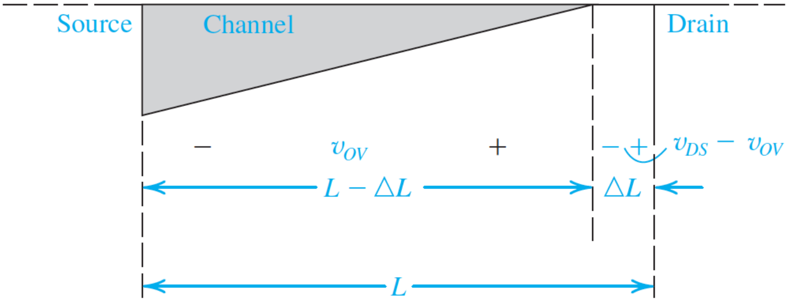

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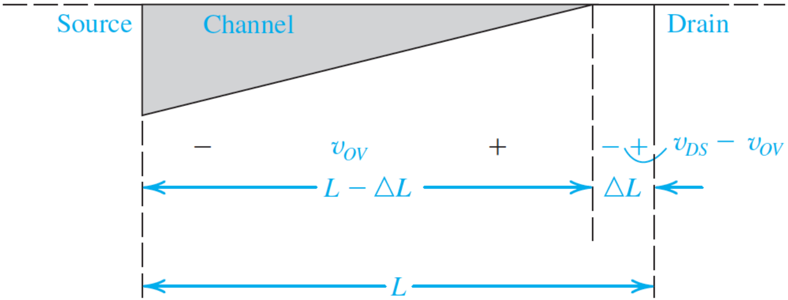
Finite output resistance in saturation

- As the channel length becomes shorter, the electric field, which is proportional to vDS/L, becomes larger.
- Since i_D is inversely proportional to the channel length, i_D increases with v_{DS}.



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- In reality, the drift current increases, and i_D increases with increasing v_{DS}

$$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2(1 + \lambda v_{DS}) \quad (4)$$

- λ is a device parameter with the units of V^{-1}, the value of which depends on manufacturer's design and manufacturing process. λ is much larger for newer tech's
- The value of λ depends both on the process technology used to fabricate the device and on the channel length L.
- In short, we can draw a straight line between V_A and saturation.

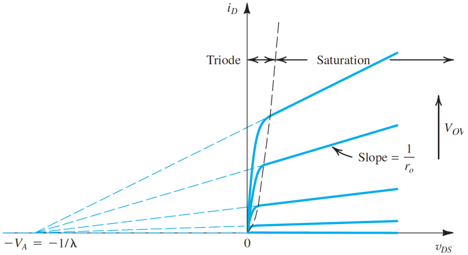

FIG 3. Early Effect—Finite Output Resistance
Effect of v_{DS} on i_D in the saturation region. The MOSFET parameter VA depends on the process technology and, for a given process, is proportional to the channel length L.

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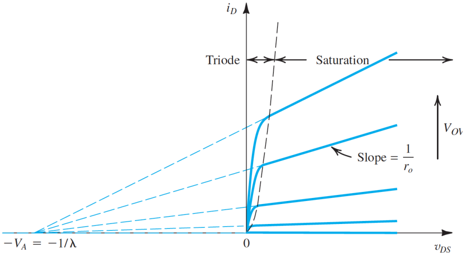

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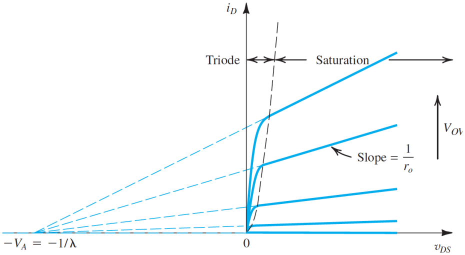

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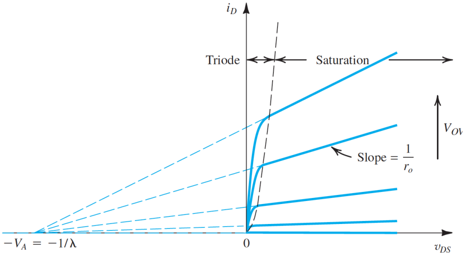

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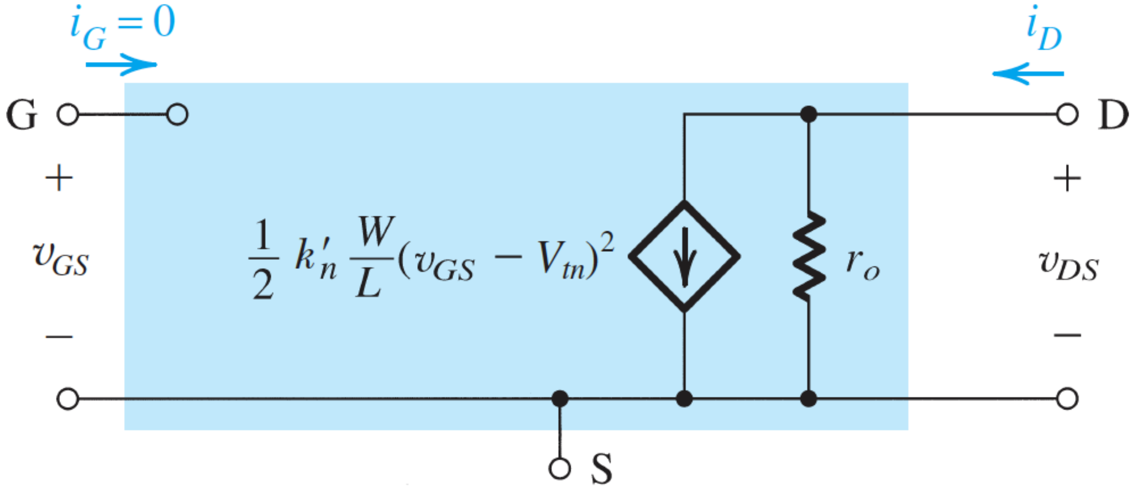

FIG 4. Large-Signal Equivalent Model of the n-channel MOSFET in saturation, incorporating the output resistance r_o. The output resistance models the linear dependence of i_D on v_{DS} and is given by Equation (4). Please note the addition of finite output resistance r_o.

Defining the output resistance

Note that r_o is the 1/slope of i_D vs v_{DS} curve

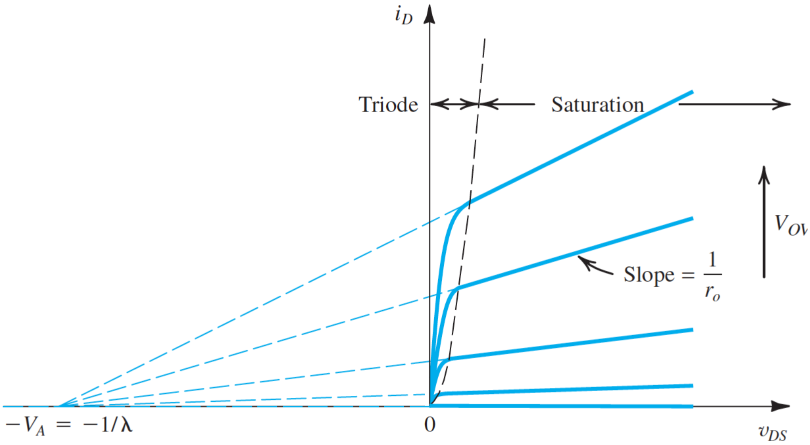

FIG 5. Early Effect—Finite Output Resistance

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- Note that r_o is the 1/slope of i_D vs v_{DS} curve

$$r_o \equiv \left[\frac{\partial i_D}{\partial v_{DS}}\right]^{-1} \tag{5}$$

- Combining Equation (4) and Equation (5), we have

$$\begin{aligned}\frac{\partial i_D}{\partial v_{DS}} &= \frac{\partial}{\partial v_{DS}}\frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS}-V_{tn})^2(1+\lambda v_{DS}) \\ &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}v_{ov}^2\lambda\end{aligned} \tag{6}$$

- Thus, the output resistor is defined as shown in Equation (7)

$$\begin{aligned}r_o &= \left[\lambda\frac{k_n'}{2}\frac{W}{L}(V_{GS}-V_{tn})^2\right]^{-1} \\ &= \frac{1}{\lambda i_D} \\ &= \frac{V_A}{i_D}\end{aligned} \tag{7}$$

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